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IMPACT OF INQUIRY APPROACHES TO MATHEMATICS TEACHING ON THE DEVELOPMENT OF SKILLS TO ANALYSE AND INTERPRET RELATIONSHIPS BETWEEN VARIABLES

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Abstract

The development of inquiry skills to analyse and interpret data from tables and graphs, identify, understand and use relationships between variables represents an important goal of mathematics teaching. Our research is currently focused on the development of high school students' inquiry skills in mathematics teaching and diagnosing the quality level of their acquisition. The first phase of the research was oriented on the development of innovative methodical and learning materials based on inquiry approaches to mathematics teaching. In the paper, we present interactive learning activities implemented in the system Geogebra, which enable students to analyse graphs of functions, formulate and test hypotheses, express relationships between variables and generalize mathematical findings. As a part of our research we conducted a pedagogical experiment in six Slovak high schools focused on application of inquiry approaches to mathematics teaching. The main purpose of the pedagogical experiment was to assess the effectiveness of inquiry-based mathematics teaching in the development of inquiry skills to analyse and interpret relationships between variables. The levels of the development of the inquiry skills were diagnosed using pre-test and post-test. The pre-test was answered by 332 students from 1st or 2nd grade groups (fifteen and sixteen year old students) of six high schools. The post-test has been given to the same groups after applying innovative teaching methods based on the inquiry approaches to learning. The post-test was answered by 288 students. The basis of quantitative analysis of the test results was a statistical testing of the hypotheses that our innovative teaching method induces a significant improvement in selected inquiry skills.

Keywords

Mathematics teaching, inquiry skills, interactive learning activities, pedagogical experiment, pre-test, post-test.

Introduction

Sciences, mathematics and technical disciplines have key importance for the continuing acceleration of scientific and technical development of the society. Despite the importance of these disciplines for the development of society interest of young people in learning mathematics and science falls and the number of students that wish to study science and technical universities decreases. Many studies have highlighted an alarming decline in young people's interest for learning of science, mathematics and informatics. A reversal of school practice from mainly deductive to inquiry-based methods provides the means to increase students' interest in science and mathematics [Rocard, 2007]. Inquiry-based teaching encourages curiosity and observations followed by experimentation, modelling, problem solving, explanation and justification of findings. Previous research has shown that inquiry-based mathematics teaching enhances students' understanding, mathematical thinking and problem-solving skills [Hähkiöniemi, 2013].

Teachers should try to find ways to enhance students' motivation, to make the education process closer to real life and to more excessively connect teaching of mathematics, physics and informatics. One of the ways to innovate the educational process is the application of inquiry approaches to teaching. Inquiry in education is, just like scientific research, aimed to ask questions, to find and justify answers. Inquiry-based teaching offers a potential for developing the students' inquiry skills. Young people should acquire skills to gather and analyse data, interpret results, conclude, justify and generalize conclusions in mathematics and science teaching.

The official documents characterizing the strategy of mathematics education in the European Union (Eurydice, 2011) emphasizes the importance of the development of mathematical and digital competencies for a full and active integration of young people in the modern knowledge society. ICT can play an important role in creating learning environment supporting students' inquiry and raising their motivation. They allow creation of an experimental, interactive and dynamic environment stimulating active learning. Successful integration of ICT into mathematics teaching requires a development of learning activities for students and various supports for teachers. In our experiment, we primarily used the dynamic geometric system Geogebra to create interactive learning activities for investigation of functional dependencies between quantities. Geogebra integrates algebraic and geometric tools and it brings new means to visualize and investigate mathematical structures and relationships in mathematics teaching [Hohenwarter, 2007]. Geogebra can help students to explore, conjecture, construct and explain mathematical relationships. It allows generating appropriate examples and counterexamples from which students can be asked to detect patterns, make conjectures, and develop arguments [Davis, Fonger, 2015].

Selected results of international testing mathematical literacy

Results of several international examinations show a decreasing level of mathematical literacy of students not only in Slovakia. In recent years, student achievement in mathematics is assessed through two large international surveys, namely TIMSS and PISA. The TIMSS 2011 [Galádová, 2015] study involved more than 65 countries worldwide. The results of the assessment offer a complex view of the learning outcomes of students in the fourth grade of elementary schools in reading literacy, mathematics and science. To illustrate this we chose a task involved identifying the relationships between data displayed in a graph.

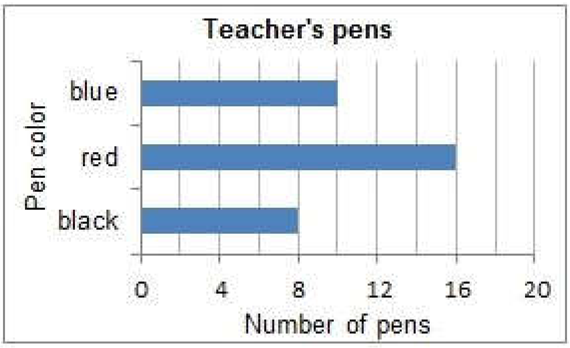

Fig. 1: Interpretation of relationships between data in the graph

The graph shows the amount of blue, red and black pens that the teacher has on his desk. How much is more red pens than black pens?

The problem was correctly solved by 45,4% of Slovak students on average. The achieved result falls considerably behind the average result of the OECD countries which is 61,7%.

The international OECD PISA study focuses on measuring and comparing learning results of 15 years old students of the respective schools in reading, mathematical and scientific literacy. In year 2012 the examination was oriented to mathematical literacy, and the main focus was on testing the ability to solve problems and on financial literacy [NÚCEM, 2013]. To illustrate the application of graphic representation of data in examining the relationships between data we chose a part of the exercise "charts". New CDs by bands 4U2Rock and The Kicking Kangaroos were released in January. They were followed in February by CDs by bands No One's Darling and The Metalfolkies. The provided graph depicts the sales of the CDs of the respective bands from January through June. The Kicking Kangaroos' manager is worried, because the number of sold CDs has been decreasing from February to June. What will be the approximate sales volumes in July if the same negative trend will continue?

A) 70 CDs B) 370 CDs C) 670 CDs D) 1340 CDs

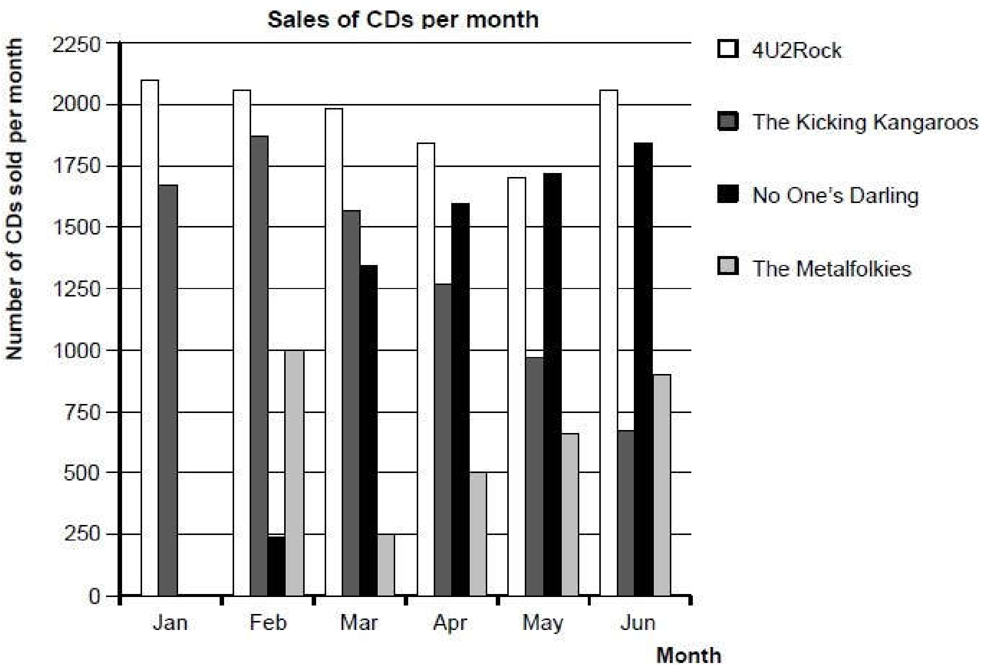

Fig. 2: Sales of CDs during the reporting period

The correct answer is 370 CDs. The average percentage value of correct answers by Slovak students was 68,5%. It is also another case in which the results fall behind the results of other countries, since the average success of the students from OECD countries was 76,7%.

The project focused on assessment of the effectiveness of inquiry-based teaching

Efforts to implement inquiry approaches to science and mathematics education are reflected in the wide range of projects supported at the level of the European Commission or by national research agencies. Researchers at our faculty participated in solving international project ESTABLISH and SAILS. The results and experience of dealing with these projects are used in the project supported by the Agency for the promotion of research and development which is aimed at the research on the efficiency of innovative teaching methods in mathematics, physics and informatics education. The main goal of the project is assessment of the impact of inquiry-based teaching on the development of students' inquiry skills. Two hypotheses were defined according to the specified goals. A natural pedagogical experiment was used for the verification of hypotheses. The main focus was concentrated on the assessment of inquiry-based teaching impact only in selected groups in high schools. The first phase of the project solving was focused on designing and developing innovative lesson plans and teaching materials for applying inquiry approaches to the teaching of mathematics, physics and informatics at high school. According to the study of school education programs, we selected topics that were corresponded to the planned timetable of the experiment.

Lack of skills and students' misconceptions, which we specified using the results of international measurements and research studies (for example Marshall, 2013), have been taken into account in designing lesson plans enabling the application of inquiry approaches to teaching mathematics. An important part of the development of innovative learning materials for inquiry-based teaching was a preparation of interactive learning activities. In the article, we focus on a description of selected inquiry activities for exploring relationships between variables and the use of functional dependencies for problem solving. Interactive activities created using Geogebra support students to explore graphs of functions, to make and understand connections between representations, construct hypotheses, and explain observed patterns. Linking various representations is an important domain in the understanding of function [Ronda, 2015]. Progression involving linking representations can begin in working with linear functions, when students can first make connections between a graphical and verbal representation and then connections between graphical and symbolic representation. Each of the three representations can highlight a particular property of the relationship between variables. We chose an activity based on real situation: *A litre of gasoline is sold for 1,58 EUR in town A. A litre of gasoline of the same kind is sold for just 1,28 EUR in town B which is 30 km away from town A. It is worth to a man living in the town A go to refuel in the town B if their car has an average gas consumption of 6 litres per 100 km?*

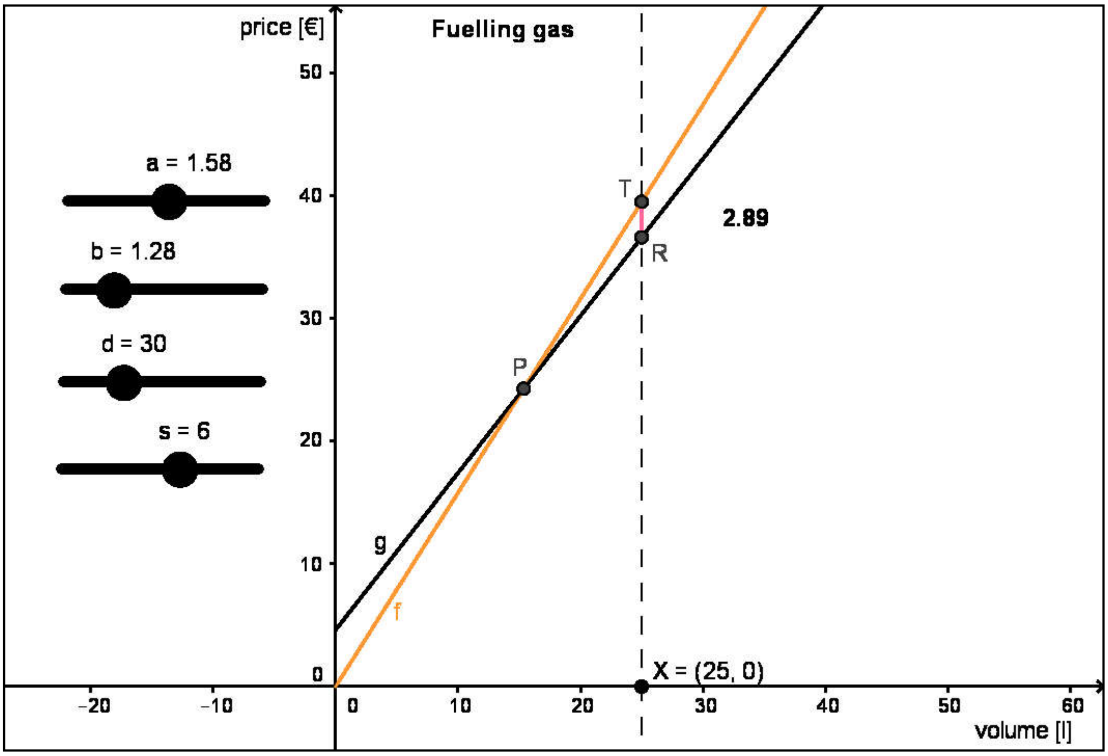

Fig. 3: Graphical model of a real situation

Students should first create a table and a graph describing the price for refuelling in the town A. Then they should decide how the graph of explored dependency would change, if the driver refuels gasoline in the town B, while considering the lower gasoline price in calculation of the price for the travel to the town B. The students would be given the dynamic construction displayed in figure 1 only after understanding the described situation. Then they would solve the following tasks:

1. Describe verbally the functional dependencies represented by graphs of functions f, g.

2. Characterize the meaning of the intersection of graphs of functions f, g.

3. Find out the lowest amount of litres of gasoline which should the driver refuel in the town B so as to be worth it if the price for the gasoline in the town B dropped to 1,20 EUR.

4. Characterize the meaning of the number displayed in the graphic window to the right of the graph (by changing the position of the point X).

5. Describe the effect of individual variables on the amount of saving for refuelling in the town B.

6. Use function formulas for f, g to express the amount of saving for refuelling in the town B.

Solved task was focused on the calculation of the price of gasoline consumed by moving car. Motion tasks may be appropriate used to investigate the dependencies between variables, for example the relationships between the moved distance or the speed of objects and the time.

The applet http://demonstrations.wolfram.com/FillingAContainerDefinedByACurve/ offers interesting theme to model the dependencies between variables. The applet is located on the portal demonstrations.wolfram.com by the company Wolfram Research, which offers a variety of interactive demonstrations useful in the teaching of mathematics and science. Start-up of interactive demonstrations requires installing the free available program Wolfram CDF Player.

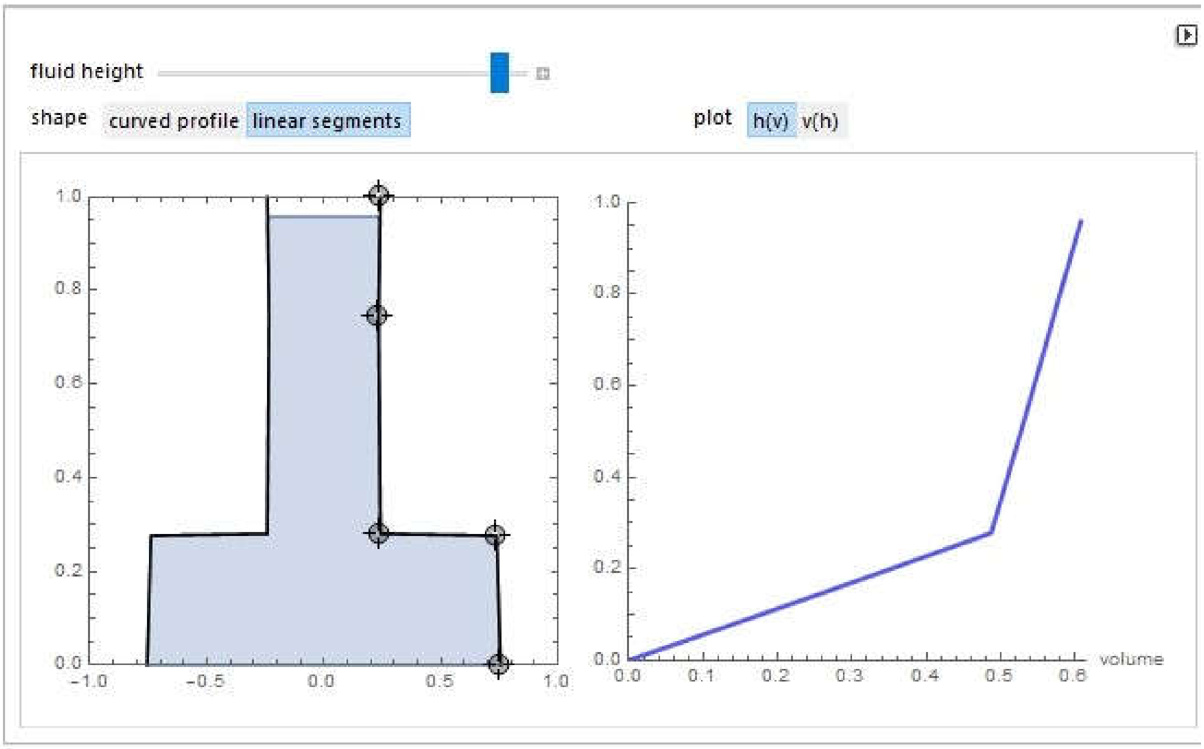

Fig. 4: Modelling the dependence between quantities

The described applet allows modelling uniform filling water into containers of various shapes and simultaneous construction of a graph of the dependence between water level and the volume of water in a container. Since the water flows uniformly into the container, investigated dependence can be interpreted also as the dependence of the water level height on the time. The shape of the container can be modelled by straight lines or with curves. The container in figure

4 is made up of two cylinders with different diameters. The graph shown at right has been built over the simulation run.

At the beginning, the teacher should explain to students in classroom described phenomenon using the example of dependence between the water level height and the volume of water for the container of cylindrical shape. The teacher should require students' predictions about the character of a graph showing the investigated dependence. The use of interactive demonstration would only follow after explanations of arguments to justify the shape of the graph. Then the teacher would ask the students to justify shapes of graphs for cylindrical containers with different diameters and for the container shown in figure 4. Investigation of more difficult dependencies for containers with more complicated shapes could follow in the next phase of teaching. Students could solve the problem: what shape should have a container in order to a graph of the dependence between the water level height and the volume of water increases faster than a linear function. One of the possible solutions is shown in figure 5.

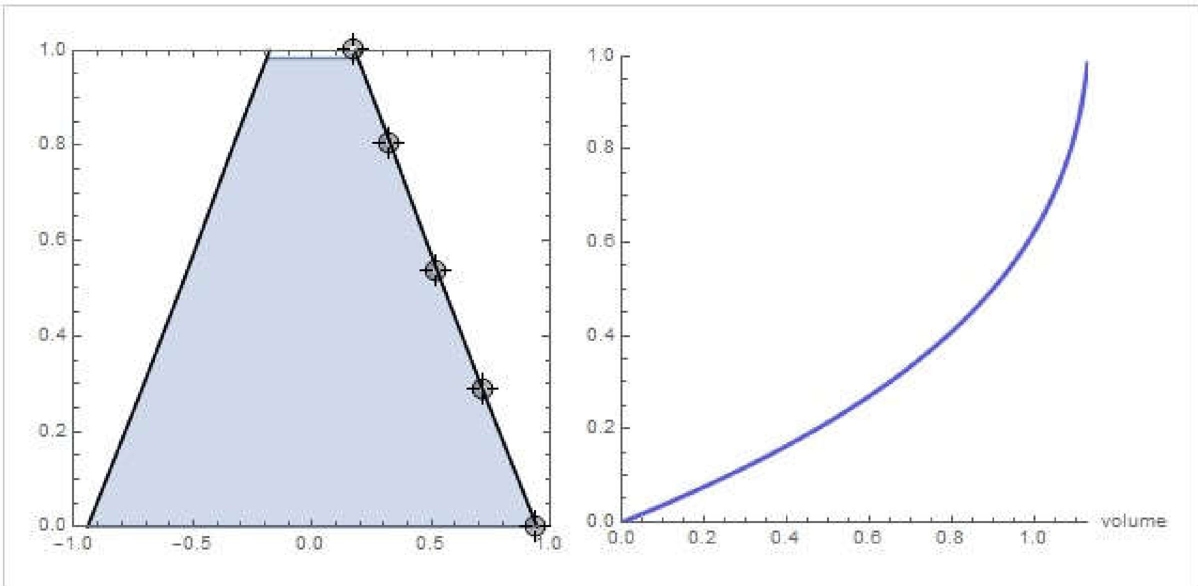

Fig. 5: Modelling of a nonlinear dependence between quantities

Since the diameter of the container is gradually reduced water level height will still grow at greater values, not at the same values as by the linear function. Finally, students could construct graphs of the dependence between water level height and the volume of water in a container for different shapes of containers on paper that would show the teacher on the blackboard. Students should also solve the opposite task and they would suggest the shape of the container in accordance to given graph. Interactive demonstration can be used to confirm or refuse students' solutions.

Graphs of functions on six cards (see figure 6) was given the students in the activity focused on analysis of graphs of functions. Students should split cards to two groups with the same number of cards. After allocation of the graphs of functions using their symmetry with respect to the y-axis or to the beginning of the coordinate system, students are asked to find function formulas which graphs have one of listed properties. The students could use the system Geogebra for the construction and analysis of graphs of functions. The teacher should also require from students the generalizations of their findings. For example, if students assign the quadratic function $y = x^2$ to functions whose graphs are symmetrical with respect to the y-axis then the teacher should lead students to conclusion that the functions which formulas can be expressed in the form $y = ax^2 + c$, where a, c are real numbers and $a \neq 0$, have required property.

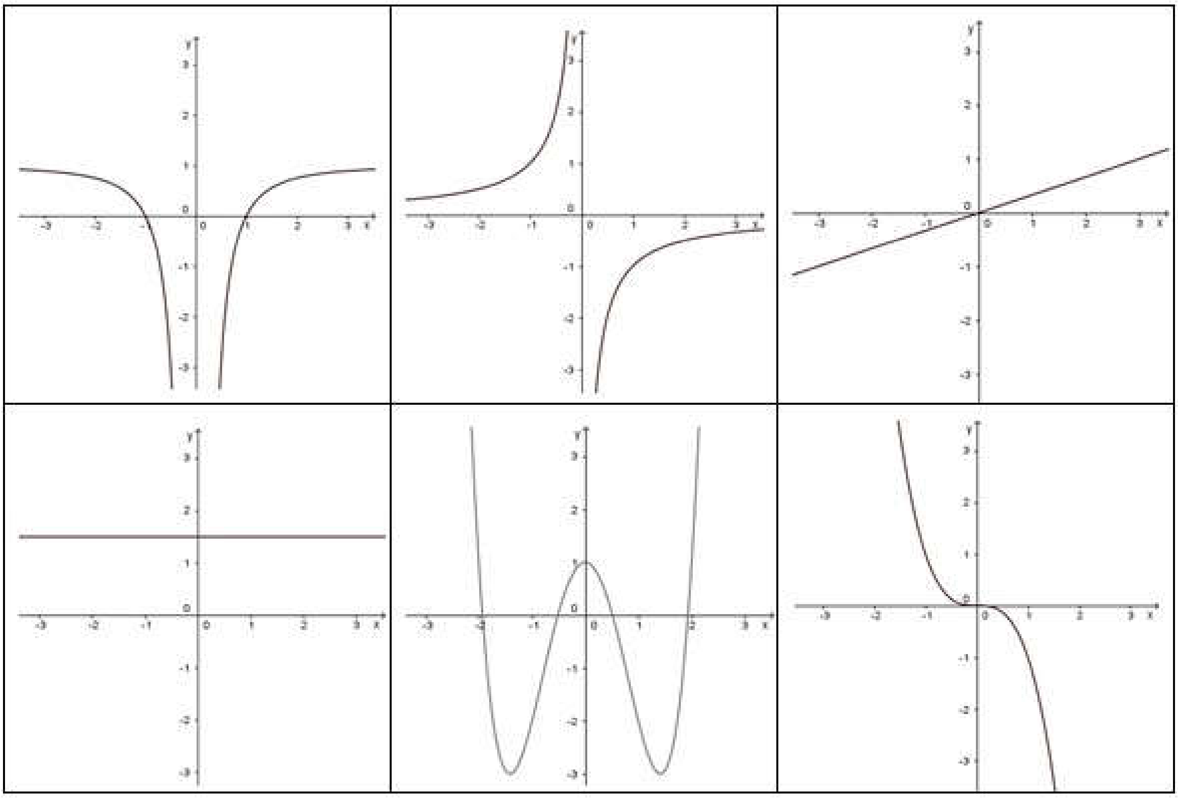

Fig. 6: Graphs of even and odd functions

The power function is convenient type of functions to investigate even and odd functions. Students should generalize claim about power functions in regard to parity of their exponent. Students should find out by the investigation of function properties that the classification of functions according to the parity is not dichotomous and there are functions which graphs do not have any of listed properties. Finally, the teacher could give students following activity for independent inquiry after introduction of the terms odd and even function: *There are certain rules for the sum or the product of two odd or even numbers. Investigate, whether the analogical rules hold for the sum or the product of two odd or even functions. Illustrate your findings with appropriate examples and find arguments to justify the discovered rules.* Students can simply construct graphs of sum and product of two functions and observe their properties using the system Geogebra. The definitions of the functions f, g in input line allow to get a graph of the function $h = f + g$ or $h = f \cdot g$ by writing of these formulas into the input line. Proving discovered findings should be based on the use of the definitions of even and odd function.

Testing a level of the development of selected inquiry skills

The created methodical and learning materials were tried in real school conditions at six partnership high schools. Experience and suggestions from the teachers were incorporated to the learning materials. The final versions of the learning materials were used in pedagogical experiment conducted in the school year 2015/2016. Design of the experiment can be expressed using the following scheme [Reid, 2006].

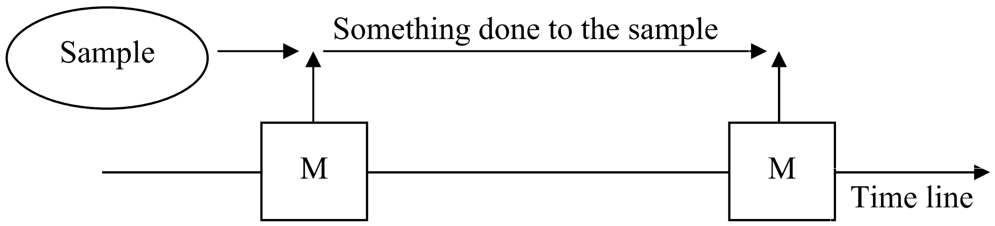

Fig. 7: Structure of the pedagogical experiment

The sample for the experimental teaching consisted of two chosen groups in the first and second class at the high schools. The teachers applied inquiry approaches to teaching of selected topics in mathematics, physics and informatics during October to March. The block in the scheme labelled as M represents a measurement with a test or a questionnaire. As an assessment instrument in the pedagogical experiment was selected a paper and pencil test to measure the level of development of inquiry skills of the students. The main goal of the pedagogical experiment was to evaluate the effect of the innovative methods of teaching on the development of selected inquiry skills of the students.

Two hypotheses were defined in the pedagogical experiment.

H1: Unsystematic and inconsistent use of inquiry approaches to teaching of mathematics, physics and informatics causes low level of the development of the selected inquiry skills of students in high schools.

H2: Systematic and coordinated use of the innovative teaching methods of mathematics, physics and informatics based on the application of inquiry approaches to teaching has a positive effect on the development of the selected inquiry skills of students.

50% performance success in the acquisition of standard knowledge and skills is often regarded as the limit of low level of success. To characterize a low level of development of inquiry skills we set the performance success of the students to 40%.

Several researchers have tried to develop assessment tools to measure students' inquiry skills. Alonso and Aschbacker [Lou et al., 2015] created a test for assessment of inquiry skills in the laboratory conditions in 2004. The test also contains various inquiry activities and requires students' experimental operations. Duration of the test was average of 2.5 hours. This was reason that the test was used mainly for research purposes. Orion and Kali [Lou et al., 2015] created the test developed within Earth science content for assessment of specific inquiry skills in 2005. The test contained twenty tasks in the multiple-choice form and it was suitable for use in the classrooms. Wenning [Wenning, 2007] created the test containing forty tasks in 2007. Each question included four options for answer of which only one option was correct. After the use of the test in real school conditions, the author admitted that closed tasks are not best suited for the measurement of specific inquiry skills.

Two measurements of the level of the selected inquiry skills development were planned in the pedagogical experiment (see figure 7). It was necessary to create the pre-test and the post-test for these measurements. Wenning's standardized test was our starting point for creation of the tests. The first phase of the pre-test development was focused on creating a database of mathematics, physics and informatics tasks suitable for measurement of the level of the selected inquiry skills. After selecting the inquiry skills which should have been primarily developed in the teaching of mathematics, physics and informatics, the first version of the pre-test was created and it was examined in three classrooms at three high schools. The final version of the pre-test was created after analysing the results of the trial run and taking into account the suggestions of the teachers which evaluated the difficulty of the tasks and suitability of the task for measurement of specific inquiry skills. Results of the pilot testing led to a reduction in the number of tasks in the pre-test. Whereas the priority requirements included the limitation of time to complete the pre-test for one lesson, the final version of the pre-test contained twelve tasks. The closed items in the pre-test included five options for answer among which two correct answers could be placed. Two semi-closed items required also reasoning answer choices. Two items of the pre-test were opened-ended. From the classification of the inquiry skills [Van den Berg, 2013], we focus on the evaluation of the development level of the inquiry skill to analyse and interpret the relationships between variables. We selected two tasks from the pre-test focused on testing the skill to interpret data from a graph (task 1) and to express the relationship between variables in the symbolic form (task 2).

Task 1: The presented graph displays the dependency of the distance that runners A, B ran in the race on the time. Based on the graph, decide which of the following statements is true.

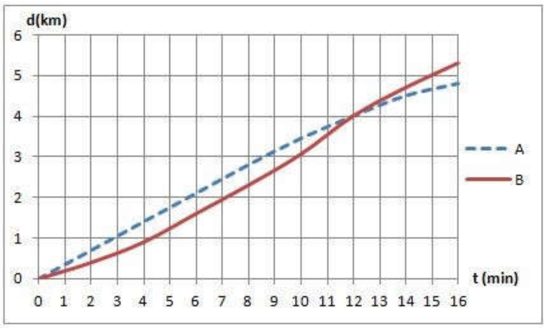

Fig. 8: Graph of dependence between variables

a) The runner B ran the first kilometre earlier.
b) The runner A ran for the first 10 minutes of more than 4 km.
c) The runner A caught up with the runner B 4 km from the start.
d) The runner A had higher average speed than the runner B during the first 16 minutes.
e) The runner B was running faster than the runner A since the end of the 10th minute until the end of the 11th minute.

The correct answer in the task 1 is the option e).

Task 2: Peter pays 18 € per night in a camp on a trip to the mountains. Since he often camps out in a national park, this year he bought a season-ticket for 70 €, which allows him to obtain the 50 % discount for 24 nights in a camp for the entire year. Let *x* be the number of nights that he spent in the camp this year. Which of the following equations could we use to calculate the total amount s of overnight accommodations at the camp for the entire year, if we know that Peter spent in camp more than 24 nights in this year?

a) $s = 0{,}5 \cdot 18x + 70$

b) $s = 0{,}5 \cdot 18 \cdot 24 + 18\,(x - 24) + 70$

c) $s = 18x - 0{,}5 \cdot 24x$

d) $s = 18x - 0{,}5 \cdot 24x$

e) $s = 18x - 0{,}5 \cdot 18 \cdot 24 + 70$

The task 2 has two correct answers expressed in the options b) and e).

The students in the experimental groups were given a post-test to measure the level of the development of the selected inquiry skills at the end of the pedagogical experiment. When creating the post-test, we considered two variants. The first variant involved including equivalent tasks containing different numbers which would base on the same or very similar context. The second variant involved including tasks based on the description of different situations. The deciding factors in choosing a variant of the post-test were fairly long period between the pre-test and the post-test (6 months) and a problematic comparing of the difficulty of the tasks. We decided for the first variant and we included only slightly modified tasks in the post-test. The following tasks in the post-test corresponding with the tasks selected from the pre-test:

Task 1*: The presented graph displays the dependency of the distance that runners A, B ran in the race on the time. Based on the graph, decide which of the following statements is true.

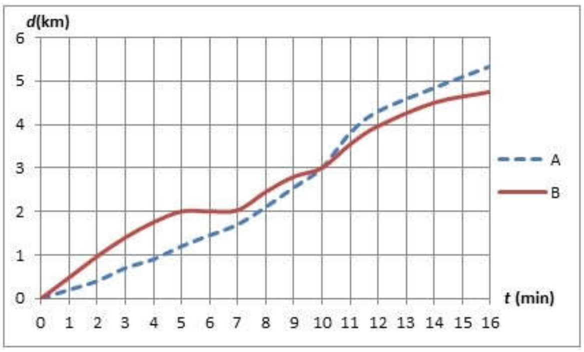

Fig. 9: Graph of dependence between variables

a) The runner B was running faster than runner A since the end of the 8th minute until the end of the 10th minute.
b) Both runners had during the first 16 minutes at least one minute break, when they were resting.
c) The runner B caught up with the runner A 3 km from the start.
d) The runner A had higher average speed than the runner B during the first 12 minutes.
e) None of the previous statements is true.

The correct answer in the task 1 is the option d). The assessment of the solution of the tasks 1 and 1* was binary. Student's percentage of achievement could be 0% or 100%.

Task 2*: Camila goes to a gym, in which one session costs 4 €. Since she exercises regularly, she paid a membership fee of 50 €, that allows her to get 50% discount for 30 sessions in the gym before the end of the year. Let *x* be the number of sessions, that Camila attended in the gym before the end of the year. Which of the following equations can be used to express the total sum *s* for sessions in the gym before the end of the year, if we know that Camila attended more than 30 sessions before the end of the year?

a) $s = 4x - 0{,}5 \cdot 4 \cdot 30 + 50$

b) $s = 4x - 0{,}5 \cdot 30x + 50$

c) $s = 4x - 0{,}5 \cdot 30x$

d) $s = 0{,}5 \cdot 4 \cdot 30 + 4(x - 30) + 50$

e) $s = 0{,}5 \cdot 4x + 50$

The task 2 has two correct answers expressed in the options a) and d). When evaluating the results of tasks 2, 2*, student's success was 100% only in the case if the student chose only two answers and both answers were correct. If the student is chosen only one correct answer his percentage of achievement is 50%. If the student is chosen the right answer in combination with the wrong answer his/her percentage of achievement is 0%.

Results and Discussion

Fourteen groups of the first and second classes in partnership high schools were participated in the pedagogical experiment. The average achievement levels of individual groups (signs of schools are stated in brackets) in the presented tasks are given in the table 1.

Group	Class	Specialization	Pre-test			Post-test		
			Count	1	2	Count	1*	2*
Kvinta (A)	1	M	27	55,56%	44,44%	26	57,69%	53,85%
1.A (J)	1	G	29	44,83%	36,21%	27	55,56%	37,04%
2.NB (O)	1	B	24	29,20%	25%	15	13,33%	30%
1.C (P)	1	L	30	36,67%	31,67%	25	24%	36%
1.A (S)	1	G	24	16,67%	18,75%	19	31,58%	36,84%
2.E (T)	1	B	24	41,67%	41,67%	22	22,73%	45,45%
Sexta A (A)	2	M	15	46,67%	50%	14	50%	57,14%
Sexta B (A)	2	M	22	36,36%	38,64%	18	27,78%	36,11%
2.D (J)	2	G	32	37,50%	34,38%	31	48,39%	41,94%
3.NB (O)	2	B	12	16,67%	25%	10	10%	20%
2.D (P)	2	G	30	36,67%	43,33%	27	29,63%	42,59%
2.B (S)	2	G	19	42,11%	34,21%	16	100%	50%
2.C (S)	2	G/L	26	15,38%	36,54%	24	25%	43,75%
2.C (T)	2	G	18	27,78%	27,78%	14	35,71%	46,43%

Tab. 1: Summary results of the pedagogical experiment

Groups are differentiated by class and specialization: G – general, M – mathematical, B – bilingual, L – language. Results in the table shows that the percentage success of students in solving the corresponding tasks in the post-test in the majority of groups in the first class was higher than in the pre-test. The groups 2.NB, 1.C a 2.E were worse in the solving the task 1* compared to the task 1. The situation in the second class was slightly worse. Three groups of students (Sexta B, 3.NB, 2.D) received worse results in the post-test by solving both tasks.

A more detailed analysis of student solutions of both tasks in the pre-test and the post-test was the starting point for finding the reasons of the above findings. Analysis of the pre-test results shows that students often chose by solving task 1 the incorrect answer c) (15,4%) and the combination of the answer c) with another answer appeared in 32,6% of students. The reason of this error could be most likely inconsistent interpretation of the data from the graph. Students most often chose from the incorrect answers the option d) (17,5%) and the combination of the answer d) with another answer appeared in 33,1% of students. This finding could point at the use of incorrect connection between the average speed of runners and mutual position of graphs. In that the graph representing the movement of the runner A was above the graph representing the movement of the runner B three-quarters of the observed period of time, some students incorrectly concluded that the runner A had higher average speed than the runner B in the first 16 minutes.

We oriented the correct answer just on average speed in the formulation of the corresponding task 1* in the post-test. 38,9% of students chose the correct answer d) and the combination of this answer with another incorrect answer appeared in 49,4% of students. Given the fact that students of six groups reached the worse results in solving this task in the post-test than the pre-test, can be assumed that the formulation of the task in the post-test influenced obtained results in some detail. 13,9% of students chose in the post-test incorrect answer c), which corresponds to the incorrect answer c) in the pre-test. The combination of this answer with another incorrect answer appeared in 24,7% of students. From these results it can be concluded that students were not improved significantly in the analysis of graphic information.

When solving task 2 in the pre-test many students chose only one correct answer. 39,8% of students chose only the correct answer b), 20,5% of students chose only the correct answer e). 4,8% of the students chose both correct answers. We can assume that some students did not analyse other answers after finding a correct one. However, bigger group of students chose the correct answer b) can also relate to the fact that this answer represents the processual approach to solving the problem. Answer e) on the other hand represents more complicated method in which it is necessary to subtract the total discount for 24 nights from the total full price fees.

The described assumptions were proven to be true in regard to the results of solving the task 2* in the post-test. The answer representing the processual approach to solving the problem was listed as d), therefore after another correct answer that represents the full price expenses which is listed as a). 46,9% of students chose only the correct answer d), 23,3% of students chose only the correct answer a). The students could find the equivalence of these answers also using simplification the right sides of the equations. We assume that the students did not use this possibility as it is shown by the fact that only 6.9% of students chose both of the correct answers. These results show that students' inquiry skill to express the relations between variables using symbolic notation improved significantly compared with the skill to interpret information from graphs. Statistical testing also confirmed this finding.

Summary of success by class, specialization and gender found using statistical tests is presented in the table 2.

Class, gender, specialization	Pre-test		Post-test		Results	
	1	2	1*	2*	1 - 1*	2 - 2*
1	37,98%	33,23%	36,57%	40,67%	Same	Higher
2	32,76%	36,78%	40,91%	42,86%	Same	Higher
Male	44,16%	38,96%	46,62%	44,74%	Same	Higher
Female	25,84%	31,46%	32,26%	39,35%	Same	Higher
G	32,02%	33,71%	44,94%	42,09%	Higher	Higher
B/L	29,32%	32,76%	20,83%	37,50%	Same	Same
M	46,88%	43,75%	46,55%	49,14%	Same	Same
Total	**35,25%**	**35,09%**	**38,89%**	**41,84%**	**Same**	**Higher**

Tab. 2: The evaluation of results

We used to verify defined hypotheses H1 and H2 the one-sample and two-sample Wilcoxon test with a significance level of 0,05. The hypothesis H1 was confirmed on the significance level for both monitored skills. Although from the perspective of the average achievements the skill to interpret information from graphs were improved in several groups but this result was not confirmed statistically.

More significant improvement occurred in several groups in the skill to determine and express the relationships between variables what was confirmed at the significance level of 0,05. The most significant improvement was in groups with general specialization. Significant improvements occurred in both genders but the level of acquisition of these inquiry skills is higher among boys than girls. This fact was confirmed using the two-sample Wilcoxon test. Improving in the skill to express relationships between variables in symbolic form was statistically confirmed in both classes. We have expected that differences in achievement levels of measured inquiry skills among the first and second classes will be markedly better in favour of the second class.

Achieved results in the post-test are overall lower than we expected. We see reasons in several facts caused by the real conditions in the school practice. Teachers participated in the project taught only two subjects of mathematics, physics and informatics at some schools. Some teachers did not fulfil the specified range of implementation of inquiry activities due to lack of time. Because of the diversity of curricula, teaching of some topics was time-shifted and it extended the duration of the experiment up to double time for the planned scope of the inquiry teaching. It could have an impact on the effectiveness of innovative teaching.

Conclusion

The acquisition of inquiry skills to analyse and interpret the relationships between variables is the basis for the development of the competencies to model real situations and solve real life problems. When creating methodical and teaching materials, a great emphasis has been placed on the use of graphical representation that is essential for understanding the functional dependencies between quantities. Static but mostly dynamic visualizations created using the dynamic geometric system Geogebra are very important for understanding mathematical relationships and generalization of the discovered findings. Results of our research has proven that systematic and coordinated use of inquiry approaches to the teaching of mathematics, physics and informatics has significant effect on the development of the students' inquiry skill to analyse, understand and express the relationships between variables with the significance level set at 0,05.

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